

# EFFICIENT CALCULATION OF THE HESSIAN MATRIX OF EM DATA MISFIT USING THE ADJOINT SOURCES APPROACH

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We present a general formalism for an efficient computation of the Hessian matrix of electromagnetic (EM) data misfit, which is based on an adjoint sources approach. This work is a step in the development of a quantitative resolution scheme in EM frequency-domain problems. The inverse of Hessian matrix is the carrier of resolution information and provides an avenue for better regularization of the inverse problem as well. Using the approach, one can calculate the Hessian matrix for a price of  $O(N_M)$  forward problem calls per frequency and polarization, where  $N_M$  stands for the dimension of the model parameter space. This is a substantial computational savings compared to a brute-force approach which requires  $O(N_M^2)$  forward problem calls per frequency and polarization. We also show that massive calculations of the Hessian-vector product requires only  $O(N_\Omega)$  forward problem runs where  $N_\Omega$  is the number of frequencies. The formalism is general in the sense that it allows to work with: a) responses that arise in EM problem set-ups either with natural- or controlled-source excitations of multiple polarization; b) various types of parameterizations; c) various coordinate systems. In addition, the formalism is not confined to a discrete formulation, and does not stick on a specific form of numerical solution.